**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: B

X=60 *μ* = 55 *σ* = 8

Z=(X-*μ* )/*σ* = (60-*55* )/*8 = 0.625*

the probability that the service manager cannot meet his commitment = 1-norm.cdf(0.625)

= 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:

1. False.

Probability between 38 and 44 = 0.3413

Probability older than 44 = 0.1586

The probability of employees between 38 and 44 is higher than the probability of employees older than 44. Hence the statement is incorrect.

1. True

PRobability = 0.0912

The center would be expected to attract = probability \* 400 = 36

The statement is correct.

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:

Central Limit Theorem: Any large sum of independent identically distribution random variables are approximately Normal then (X1 + X2) and (2X1) tends to have Normal distribution only If X1 and X2 are i.i.d and n is Large.

The size of the two separate sample subsets (X1 and X2) from the same source (population) that are held by 2X1 and (X1 + X2) differs. If X1 N(, 2) then, 2 X1 N(2, 4 2) X1 and X2 can be different subsets of a sample from a comparable source (population). If X1 and X2 are iid normal random variables, then (X1 + X2)N(+, 2+ 2)(2, 2 2) follows. As a result, 2X1 - (X1+X2) (2 - 2, 4 2 + 22) Every sample subset from a similar source has the same distribution, which often fits the normal distribution with a few minor parameter variations.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: D

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:

1. Range: (99.00810347848784, 980.9918965215122)
2. The 5TH Percentile of profit for the company is 17 Crore Rupees
3. (Profit2 N(7, 42)) is more likely to have a loss in any given year.